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Discovery learning in mathematics by using dynamic geometry software GeoGebra – action research

It is necessary to get involved actively in the process of acquiring knowledge to learn mathematical contents successfully. The theory of constructivism emphasizes student's individual activity in which he/she creates his/her own knowledge through his/her own experience, while the teacher prepares and shapes the environment for learning and encourages and guides students in their independent learning. The aim of this approach is for students to learn how to learn, and Polya's problem-solving model provides an answer how to perform in practice - in 4 steps: understanding the problem, devising a plan, carrying out the plan and reflection.

In teaching practice, this can be achieved by using discovery learning or inquiry based learning. In mathematics, dynamic geometry software can be of great help because of its possibilities to display the dynamism and interactivity of mathematical objects so it provides a dynamic virtual environment for research. Among such programs GeoGebra is distinguished because it's free and open source, translated into Croatian, intuitive and easy-to-use tool, combing elements of geometry, algebra, analysis and statistics. Moreover, the community of its users on the Internet is large and very active so there are more and more completed examples that can be used in everyday teaching.

There were various problems the author encountered in her teaching practice when trying to implement the ideas described above, so she opted for action research which seeks to respond to the problem: how to help students to discovery learn new mathematical terms, concepts and ideas individually by using computer and GeoGebra . This article describes the research and presents a report on the achieved action research with second grade students in High school Čazma in Čazma.

Keywords:
action research, GeoGebra, Pólya's problem-solving model, constructivism, discovery learning, dynamic virtual learning environment

Introduction

One of the aims of teaching mathematics is to teach students how to think and train them for solving problems and making decisions in their future life. Kurnik (2008, p. 53) stresses that *"the basic guidelines for the modernization of teaching are inducing and starting students' thinking and ensuring that a great deal of new knowledge is acquired by their own strengths and abilities."* (free translation by the author) The theory of constructivism emphasizes this approach, it is based on the fact that knowledge is constructed by pupils' activity and therefore the role of the teacher as a source of information decreases significantly in relation to the role of the teacher who will lead and direct the students in their learning. Knowledge cannot be transmitted from the teacher to a student in a way that information from the teacher is copied and pasted into in student's head. The student must be involved actively in learning so he/she could reach his/her own insights by individual work. The way of learning is unique for each student because he/she constructs his/her knowledge on his/her own activity based on personal experience. A teacher's role is to choose appropriate teaching forms, teaching methods, sources of knowledge and thus to shape the environment for learning and to encourage and guide students in their independent discovery of new terms, concepts and principles in the contents they are learning.

Teaching models which match up the requirements above are discovery learning and inquiry based learning (Bognar, Matijević, 2002). **Discovery learning** is an experiential learning that takes place in reality or uses a simulation. The simulation by dynamic geometry software is being used in teaching mathematics increasingly because of its possibilities to display the dynamism and interactivity of mathematical objects so it provides a dynamic virtual environment for the research. Among such programs GeoGebra is distinguished because it is free and open source, translated into Croatian, intuitive and easy-to-use tool, combing elements of geometry, algebra, analysis and statistics. Moreover, the community of its users on the Internet is large and very active so there are more and more completed examples that can be used in everyday teaching.

Main feature of **inquiry based learning** is to encourage students to come to certain findings by individual exploring with the appropriate assistance of teacher. It allows a high level of differentiation in teaching because teacher will help each student as much as he/she needs. The student will discover the meaning and relevance of information individually which will lead him/her to a conclusion or reflection on the newly attained knowledge. The aim of this approach is for students to learn how to learn, and Polya's problem-solving model provides an answer to how to perform in practice - in 4 steps:

1. **Understanding the problem** - read the task with understanding, student can try to answer the following questions: *What is the unknown? What are the data? What is the condition?*
2. **Devising a plan** - perceive pattern (rule, formula, theorem) that connects the known and unknown variables, more complex task divides into several easier.
3. **Carrying out the plan** - perform each step carefully.
4. **Reflection** - checking the results, answering questions *Can you derive the result differently? Can you see it at a glance? Can you use the result or the method for some other problem?* (Pólya, 2004).

More about how Pólya's model of problem solving is installed in GeoGebra dynamic learning environment and within students' exploring and experimental learning can be read at Bjelanović Dijanić (2010) and Karadag and McDougall (2009). The following is an excerpt from the report of action research that the author carried out with her students as they learned just to the ideas described above.

Context of research

Action research participants were second grade students in High school Čazma (there are 20 in the class 2c) and me, Željka Bjelanović Dijanić as the researcher and their Math and IT teacher. I teach Math in two classes of general high school and I teach IT subject in all grades in our school so my IT classroom is always available for us. There is one main computer (server) that serves 16 clients, LCD projector and screen in the classroom and Internet access is available to all 16 clients. Sometimes I organize students to learn Math individually or in pairs with interactive digital teaching materials.

The course of action research was followed by three critical friends:

1. PhD. Branko Bognar - assistant professor at Faculty of Philosophy in Osijek, dealing with themes related to action research[1], also a mentor for the methodological aspect of this research;
2. Šime Šuljić - Math teacher at Gymnasium and vocational school Jurja Dobrile Pazin, engaged intensely in using computers in Math teaching, developing interactive digital materials and regularly following the development of cognitive tools in the world;[2]

[1] Doctoral thesis: *Possibilities of enabling teachers to realize action research enquiries by using electronic learning processes* (mentor prof. dr. sc. Ana Sekulić-Majurec, Faculty of Philosophy in Zagreb, 2008.)
[2] Šime Šuljić is one of the translators of GeoGebra into Croatian, the author of dozens of professional articles and lectures about using computers in math teaching.

3. Željko Kralj – the principal of High School Čazma and Math teacher with a 30-years experience showing great interest in my effort to introduce changes in Math teaching.

The problem and research plan

The action research is based on several educational values: **independence** in learning, the **activity** of all students, **collaboration** between students, **motivation** for learning mathematics and **computer-assisted learning**. The full report of action research goes beyond this article so here only a part of the research is going to be presented related to the independence, the activity and computer-assisted learning.

Almost all of these values are disrupted in traditional classes where the activity of teacher in frontal teaching dominates. Students are too reliant on the teacher and most of them are not able to solve the task at home if a similar one is not resolved in the class. Only interested students are active in the class while others are waiting to copy from the board. Computers and the Internet are the media that surround students every day so it is logical to offer a computer-assisted learning as an alternative.

However, the computer is not going to improve the current situation by itself. I even tried to realize discovery learning within computer-assisted learning in previous generations of students but we encountered some difficulties and the results were not improved essentially. That is why I decided to introduce changes in my educational practice and carry out the action research in the class 2c. All of the 20 students said they wanted to participate and their parents signed an agreement statement. I expect that we are going to identify why students have difficulties in individual discovery learning using computer and GeoGebra based on systematic observation and critical questioning, and that we all together are going to overcome the difficulties in learning mathematics content with new teaching methods.

Therefore, we stated the problem of action research with the question:

How to help students to discovery learn new mathematical terms, concepts and ideas individually by using computer and dynamic geometry software GeoGebra?

A confirmation for the validity of decision to explore previously stated problem using action research, I found in the book "Research Methods in Education" (Cohen, Manion and Morrison, 2007, p. 226) where it is stated that action research can be used to explore methods of teaching – replacing traditional method with discovery method and in developing new methods of learning. According to the classification Zuber-Skerritt (see Cohen et al, 2007, p. 232) this research is emancipatory because the participants (teacher and students) feel responsible for solving the problem presented and will attempt to resolve it through a cyclical process: plan - action - observation - reflection. For being as much successful as we can be it was planned to implement the activities through three cycles as seen in Table 1.

Table 1. Research plan

<table>
<tr><th colspan="3">Aims</th></tr>
<tr><td colspan="3">1. Training students for individual discovery learning using dynamic geometry software GeoGebra.
2. Development of new and adaptation of existing digital materials needed for individual discovery learning using computers.</td></tr>
<tr><th colspan="3">Criteria</th></tr>
<tr><td colspan="3">• Students show more independence in learning new mathematical terms, concepts and ideas.
• Students come to conclusions individually.
• Almost all students are active in the class.
• Students visit Web sites with digital materials at home on their own initiative.
• Students are more satisfied on Math lessons because they work on computers more often.</td></tr>
<tr><th colspan="3">Activities</th></tr>
<tr><td colspan="3">For the first aim it is predicted that students discovery learn lecture units listed below independently (two school hours for each unit) in the IT classroom through three cycles:</td></tr>
<tr><td rowspan="3">1. cycle</td><td>Lecture unit</td><td>Transformation of quadratic function f(x)=ax²</td></tr>
<tr><td>Digital material</td><td>http://www.normala.hr/interaktivna_matematika/kvadratna.htm</td></tr>
<tr><td>Data collection</td><td>• the questionnaire 2 (scale, open-ended questions)
• students' papers - a worksheet and a short test
• photos of the class
• teacher's research diary
• critical friends' notes, e-mail messages</td></tr>
<tr><td rowspan="3">2. cycle</td><td>Lecture unit</td><td>Roots of quadratic polynomial and its graph</td></tr>
<tr><td>Digital material</td><td>create a new digital material with the help of critical friend Šime Šuljić</td></tr>
<tr><td>Data collection</td><td>• video recording and photos of the class
• notes on systematic observation (videotape)
• students' papers - written knowledge exam
• group interview
• teacher's research diary
• critical friends' notes, e-mail messages</td></tr>
<tr><td rowspan="3">3. cycle</td><td>Lecture unit</td><td>Definitions of trigonometric functions</td></tr>
<tr><td>Digital material</td><td>translate and adapt material with permission by A. Meier
http://www.realmath.de/Neues/10zwo/trigo/sinuseinf.html</td></tr>
<tr><td>Data collection</td><td>• video recording and photos of the class
• notes on systematic observation (videotape)
• the questionnaire 3 (Likert scale)
• teacher's research diary
• critical friends' notes, e-mail messages</td></tr>
<tr><td colspan="3">Duration: in November and December 2010.</td></tr>
<tr><td colspan="3">For the second aim an additional activity is predicted that teacher does independently and continuously: studying references concerning the development of interactive digital learning materials that correspond to the specifics of teaching mathematics. (Bjelanović Dijanić, 2009, Hohenwarter, Preiner, 2007)</td></tr>
</table>

The process of conducting action research

1. cycle: Transformation of quadratic function

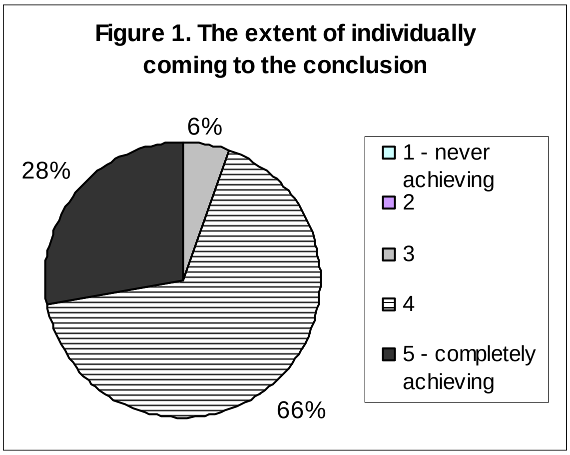
The teaching materials that we were used is on the Web of association Normala *Interactive mathematics*[3] and have been used with the previous two generations of students. The author of material is Šime Šuljić. Students themselves formed pairs according to who they like learning with. I noticed that each pair consists of students of equal ability and prior knowledge and they progressed at a pace that suits their abilities. They rarely called for a help, the instructions were clear to them. However, in the third exercise most students had difficulties: *"Read the instruction but called me because they did not know what to do (better students). Some understood how to choose the vertex, but did not understand what about the other points."* (research diary, 15/11/2010) Critical friend Željko Kralj was present at the class and he carefully observed what was happening. After the class he said this way of teaching seems very demanding for the teacher because it requires walking around the students all the time but they don't progress at the same speed at the same time so the teacher should know all the lessons through which they are going through very well. He also noticed some students really did well while another did not understand what to do (research diary, 15/11/2010).

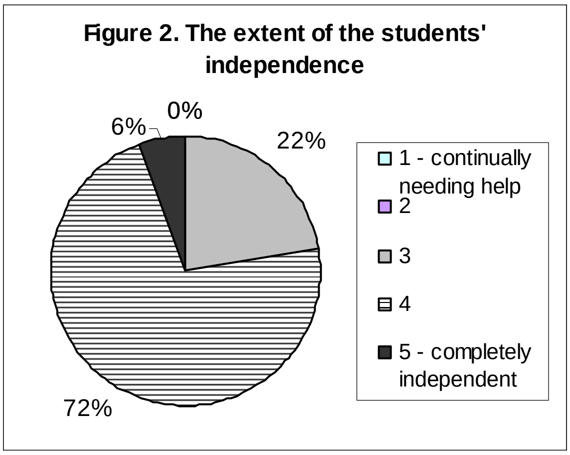
Picture 1. i 2. The computer in the class - frontally or individually (in pairs)

Picture 1. shows how we use a computer and a projector in frontal teaching and Picture 2. shows how students learn in pairs using digital materials. In the e-mail sent on 14/12/2010 Branko Bognar gave his review of these two photos: *"There is possible to see the whole class where the teacher is dominant at the first photo, while at another one students are active. On the faces of students (second photo) it is possible to observe the interest and concentration on the activities they are engaged."*

[3] http://www.normala.hr/interaktivna_matematika/kvadratna.htm

In questionnaire 2. I wanted to determine the extent of the students' independence in learning and reaching conclusions, and if they were visiting the Web site at home.

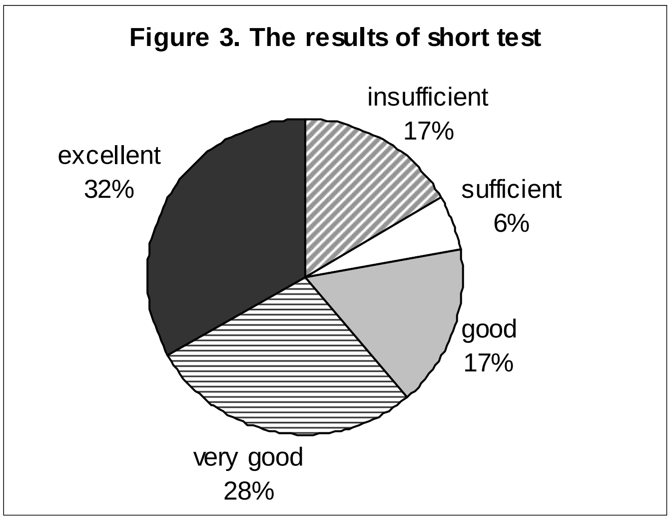




The students knew the digital material is available on the Internet, but in the class I did not emphasize it would be good to visit these Web pages at home a few more times. Still, 11 of 18 students visited the Web site at home.

Related to the worksheet, all students solved the tasks requiring short answers, and about a half of the students solved the task describing the effect of parameters on the graph. Most students had problems with the task 19 (determining the equation of quadratic function from the sketch) and the task 20 (plotting the graphs of quadratic function) so these answers were analyzed and we solved an example on the board. During the conversation several students noticed the problem: they learned using computer but were examined on a paper (research diary, 23/11/2010).

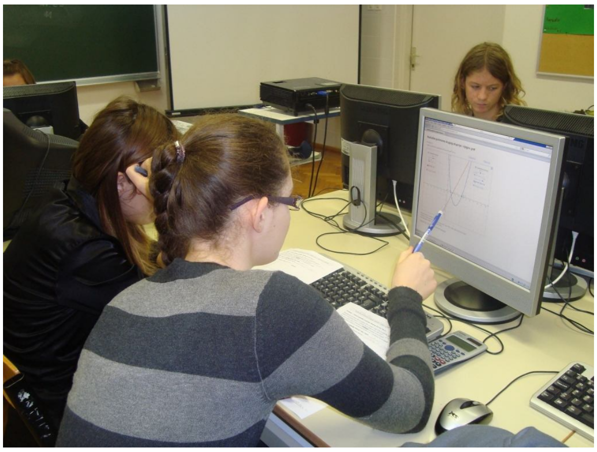
The analysis of short test containing two tasks of plotting (as in task 20) and three tasks of reading the equations from the sketches (as in task 19) showed very good results and the progress for most students:



After the first cycle reflection I submitted the results to critical friends. Željko Kralj was in contact with me and the class very often, he reviewed pupils' papers and had no remarks except some students should be neater when plotting graphs. Branko Bognar suggested to try to collect more qualitative data.

2. cycle: Roots of quadratic polynomial and its graph

I tried to make a digital teaching material for this unit myself (two dynamic applets and a worksheet). However, students had difficulties understanding the instructions at the first class in the IT classroom (23/11/2010) so Šime Šuljić helped me redesigning the material. Next time (25/11/2010) they began with the new material[4] and I recorded the situation in the class with a video recorder. After reviewing the video recording I wrote: *"Teacher walks around the class trying to be discreet, looking behind the students what they are doing and intervenes if notices students do not understand well. The students are working, they rarely call for help, rather turn to neighbors if necessary."* (research diary, 30/11/2011) We took a few photos:



Picture 3. i 4. The students work with the digital material and the worksheet

Šime Šuljić gave his review of these photos: *"The first photo shows the working material - the applet and the work of students in pairs. Obviously, they discuss their ideas. Schoolgirl right opposite is focused on the problem. On the second photo the students fill in the worksheet relying to the interactive content on the computer. It seems they occasionally check the result on the calculator. The photo tells me the working temperature is high."*

We did the remaining lecture units in the classroom, several of them frontally with GeoGebra on the LCD projector. We should also prepare for the written exam of this lecture topic. The results of the third exam can be compared with the results of the previous two:

[4] http://free-bj.t-com.hr/zbjelanovic/apleti/nultocke.html

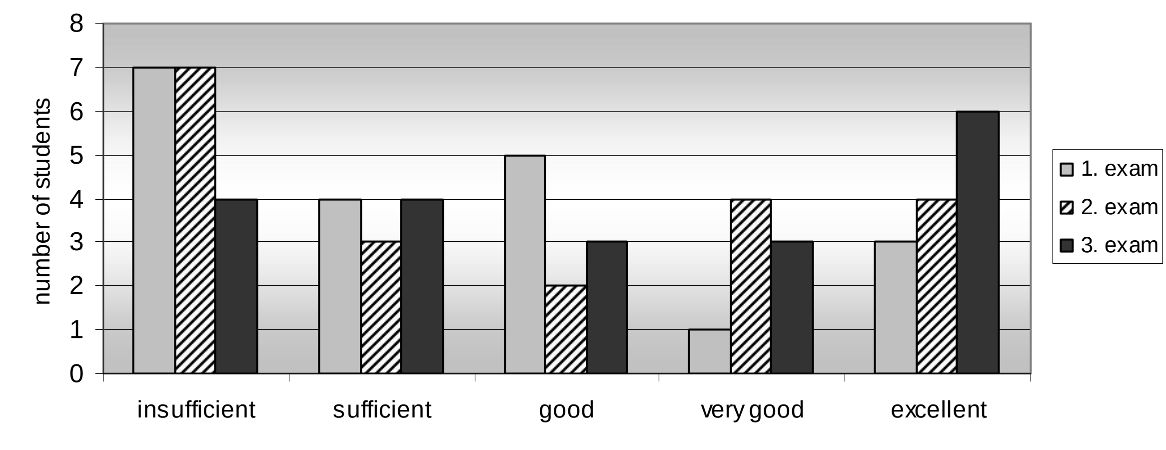


Figure 4. The results of exams in the class 2c

In both previous exams pass rate was 65% while in the third even 80%. Besides decreasing the number of negative grades, we can notice the increase of the number of excellent grades. Željko Kralj reviewed students' papers and talked to students. He was very satisfied with the results (highlighted the pass rate of 80%), but cautioned the students who wrote and drew with unsharpened pencil and drew a coordinate system without ruler.

To complete the reflection of the second cycle and to obtain students' reviews of the realization of critical-reflexive practice, we organized a group semi-structured interview.

Table 2. Group interview (taken from the transcripts and categorized)

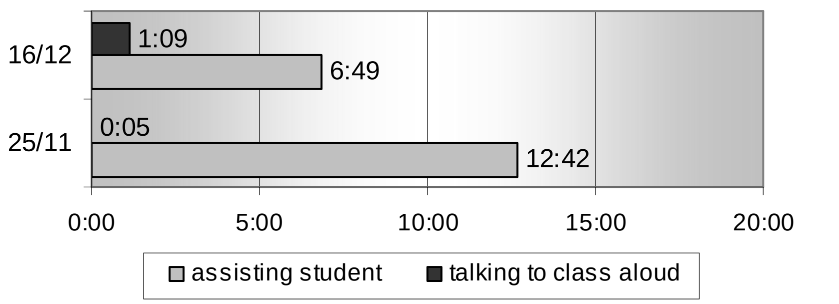
	Students' answers in the group semi-structured interview
Advantages	• interesting • it's easier to learn because of lots of examples • individually coming to the conclusion is encouraging • it stays in our memory longer • we can visualize what we do, visually remembering more • we can learn at own pace • we can repeatedly pass the same thing to clarify what is not clear to us • learning material on the Web, we can do it at home
Disadvantages	• it is not explained how you explain • the problem with understanding instructions • computer plots automatically, but we need to do it gradually • we should write the most important thing in a notebook
Suggestions for improvement	• it's better to work in pairs, we can discuss with our partner • to have a worksheet so we could write on it • this is better for practice than learning, to practice on a computer after understanding • the sliders should be more precise, not skipping, so we can check our homework • feedback, when we gave the wrong answer to show the correct

3. cycle: Definitions of trigonometric functions of acute angle

Šime Šuljić and I together adjusted the German material by Andreas Meier[5] for this lesson. I wrote to him to give us the permission for using and adopting the material in Croatian but he did not respond so we decided to proceed under the Creative Common license (share alike). We adjusted the material[6] and stated who the original author was. Meier had prepared a lot of interesting interactive exercises but their adjustment would require a lot of time, so we agreed students could practice the original German exercises.

This lesson took place on 16/12/2010 in the IT classroom. The students chose whether they wish to work in pair or alone. We recorded the class with the video recorder and the camera again. I noticed in the research diary (16/12/2010): *"We can see the improvement in students' handling learning, the exercises are motivating and students specially look forward to collecting points. They easily solve the worksheet. There were bugs in some Meier's applets - warned the students how to avoid them. Most students completed the tasks in 45 minutes so we continued by the workbook."* After reviewing the video recording I wrote: *"This time the teacher speaks more to the entire class, and therefore they call for the individual assistance less. The atmosphere is relaxing, students help each other a lot more."* (research diary, 18/12/2010) I carefully reviewed and analyzed the first 20 minutes of both recordings (until then the students have not finished yet) and separated two categories: teacher assists students (individually or in pair) and teacher talks to the class aloud. I counted how many times each category appeared and the duration.

Figure 5. The analysis of the video recordings - total duration of teachers' activities



On the first recording (25/11) the teacher assisted the students (referring to the quiet conversation between the teacher and one or two students) 28 times in the total duration of 12:42 minutes (63.5%), which means students had teacher's assistance an average 6.35% of the time, so the rest refers to individual activity. On the second recording (16/12) teacher

[5] http://www.realmath.de/Neues/10zwo/trigo/sinuseinf.html
[6] http://free-bj.t-com.hr/zbjelanovic/apleti/sinus.html

talked to the entire class aloud 9 times in the total duration of 1:09 minutes and assisted individually 17 times in the total duration of 6:49 minutes (34.08%), which is an average of 3.41% per student (because of 10 pairs).

At the end I incorporated all the questions and dilemmas that we encountered during the process and made Likert scale which will serve as a critical reflection for the data analysis and the verification for the realization of our initial educational values.

Table 3. Likert scale (taken from the questionnaire 3.)

	Statement	Strongly disagree	Disagree	Neither agree nor disagree	Agree	Strongly agree
1.	Individual computer-assisted learning enables easier processing.			20%	65%	15%
2.	Individual computer-assisted learning is more interesting then teaching in our classroom.			25%	45%	30%
3.	The computer allows me to easily come to conclusions on my own.			10%	70%	20%
4.	The computer helps me visualizing mathematical concepts.			15%	40%	45%
6.	The computer makes me think more than teaching in our classroom.			10%	70%	20%
10.	Progress at own pace is the advantage of individual computer-assisted learning.			25%	50%	25%
11.	The computer easily and automatically plots graphs so later I have problems with plotting graphs on a paper.	5%	20%	40%	30%	5%
12.	The digital material available on the Web allows me to repeat and clarify at home what I did not understand well in the school.			10%	30%	60%
15.	The feedback on the accuracy of our answers makes it easy to go through the content.			10%	75%	15%
17.	The worksheet for writing answers with pencil helps me go through the content of the digital material better.			15%	60%	25%

The interpretation of the action research

A starting point for this research was the theory of constructivism which emphasizes that knowledge is produced by the activity of student and the practice which shows it is not easily feasible in traditionally organized teaching. In practice, we noticed some students have serious problems in processing mathematical contents so we asked ourselves how to achieve the activity of all students in the class and how to encourage independence in learning. These were the initial values from which we started. As Kurnik (2008) proposes, we wanted to achieve that students come to their own knowledge using their own strengths so we opted for computer-assisted learning. However, we do not mean the using of computer in frontal teaching with LCD projector or smart board (Figure 1), we ask from everyone to work at his/her own computer (Figure 2). If we add Pólya's heuristic learning approach (Pólya, 2004), the individual discovery learning can be the one of solutions for the initial problem.

Considering we collected quite a lot of qualitative and quantitative data in the research, their synchronization will serve as triangulation process to avoid one-sided approach of qualitative data analysis so we could reach valuable conclusions.

On pictures 2, 3 and 4 it is evident that we have reached the **activity of all students** in the class. This is confirmed with the results of written exams that are much better than usual. In short test (Figure 3) one third of students were assessed with an excellent, while in the third written exam (Figure 4) 80% of students passed the exam. As advantages the students emphasize the progress at their own pace (75% of students, Table 3, statement 10) and the fact they can repeat the same procedure many times until they clarify the problem. 90% of students (Table 3, statement 6) believe the computer makes them think more than in traditionally organized teaching. We assume all of this affects higher level of students' activity in the class. Furthermore, we notice students visit Web sites with learning materials at home on their own initiative (questionnaire 2, 61% of students) and availability on the web enables them to clarify at home what they did not understand in school (90% of students, Table 3, statement 12).

The analysis of video recordings (Figure 5) indicates the **increase of independence**. We differentiate between the independence in learning process and independence in reaching conclusions. In the first cycle (Figure 1 and 2) we noted students are more independent in reaching conclusions (28%) than in computer-assisted learning in general (6%). This can be explained by the fact that students had difficulties following the instructions at first so they called the teacher for assistance or consulted other students more often. After understanding the instructions they easily come to conclusion with the assistance of computers. Students regard individual coming to conclusions very stimulating (Table 2), and this is what we aspire.

Qualitative data (photos, videos, critical friends' statements, group interview) indicate the value of computer-assisted learning. 75% of students believe this way of learning is more interesting than traditional teaching (Table 3, statement 2). It enables easier processing of the material (80% of students, Table 3, statement 1) because the computer can generate many different examples in a short period and helps with the visualization of mathematical objects (85% of students, Table 3, statement 4). The computer provides instant feedback that helps students follow the digital content (90% of students, Table 3, statement 15), and exercises with collecting points are particularly motivating.

However, we observed the disadvantages of this way of learning. Students learn by using computers but they were assessed by using written exam (on a paper). We tried to alleviate this contradiction using the worksheet that students fulfilled during studying the contents of digital material. 85% of students (Table 3, statement 17) stated a worksheet helps them to follow the content easily. Even better solution would be to develop computer-based assessment. Furthermore, we noticed some students had difficulties with plotting graphs on the paper because the computer plots graph automatically (Table 3, statement 11, Kralj's comments). We spotted the problem of understanding the instructions, and it mostly occurred with those students who usually have difficulties with understanding the task. The solution to this problem can be found in heuristic learning approach and in the art of discovery (Pólya, 2004) so it should be dealt continuously regardless of the organization of teaching.

The changes I introduced in my teaching practice within the process of action research are the result of various factors: the readiness of both the students and me to act on the basis of initial values, continuous cooperation and appreciation of everyone's opinion, the responsibility of students for their own progress, the support and advice from critical friends and the self-critical consideration of their comments. Unlike the previous generations of students, these students approached computer-assisted learning more seriously which was greatly contributed by the process of action research and the reflection after each cycle in particular.

After analyzing the collected data critical friend Željko Kralj made the general conclusion: *"The computer-assisted learning transforms student from a passive observer to become an active participant, and therefore his/her motivation for learning increases."* However, this does not mean our action research is completed, now it opens new perspectives, we spotted problems and potential solutions and opportunities for improvement. The issues that we will deal with in the future are:

- the implementation of Pólya's heuristic learning in a dynamic learning environment,
- assessing the students using computers and interactive digital resources,
- developing new and improving existing digital materials (this was the second aim that was achieved in the framework of action research, but for the further implementation of this way of teaching it is necessary to provide quality digital learning materials).

Conclusion

Discovery learning mathematics with the assistance of computer and dynamic geometry software GeoGebra using the interactive digital learning materials that are methodically and didactically designed, ensures the activity of all students in the class, increases the motivation for learning mathematics, encourages individual coming to conclusions, as well as the cooperation between students. It is very important that student learns alone or in pair on the computer trying to discover new knowledge. This knowledge is going to stay longer in his/her memory and student is going to learn how to think, he/she is going to resolve a new problem much easier and master the art of discovery.

However, its feasibility in teaching practice depends on the school conditions. It has been shown that students are willing to cooperate on implementing changes in teaching, but if Math teacher is not able to use the IT classroom for his/her classes, he/she will not be able to implement the ideas described in the paper. However, I hope the reading of this article will encourage other teachers to try to introduce changes in their teaching practices in accordance to the school conditions they work in.

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